Chapter 13

LET THE RABBI SPLIT THE PIE

A man dies, leaving more debts than assets. How should the estate be divided among his creditors? Two thousand years ago, the sages of the Babylonian Talmud addressed this question in a mysterious way—by offering a series of numerical examples, with no hint of the general underlying principle.

Take a concrete example. Suppose three creditors are owed $100, $200, and $300 respectively–a total of $600 in debts–but there is less than $600 to distribute. Who gets how much? The Talmud (Kethubot 93a) makes the following prescriptions:

1. If there is $100 to distribute, then everyone gets an equal share; that is, everyone gets $33.33.

2. If there is $200 to distribute, then the first creditor gets $50, while the other two get $75 each.

3. If there is $300 to distribute, then the first creditor gets $50, the second gets $100, and the third gets $150—the payouts are proportional to the original claims.

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Where do these numbers come from, and what should we do if there is, say, $400 or $500 to distribute? The Talmud does not tell us. But certain patterns are evident.

Apparently the rabbis reasoned that nobody can legitimately claim more than the entire estate. Thus when the estate contains only $100, the claims to $100, $200 and $300 are treated as equal. When the estate contains only $200, the claims to $200 and $300 are treated as equal (but superior to the claim of $100).

Another clue can be found elsewhere in the Talmud (Baba Metzia 2a): "Two hold a garment; one claims all, the other claims half. Then the one is awarded 3/4, the other 1/4." The rabbinical reasoning seems to have gone something like this: "Both claim half the garment, while only one claims the other half. So we'll split the disputed half equally

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and give the undisputed half to its undisputed owner." Elsewhere in the Talmud, the rabbis apply similar reasoning to settle a case where one claims all and the other claims a third.

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Now we have two principles: First, claims cannot exceed 100% of the estate, and second, we should follow the contested garment rule. With these two principles alone, we can prescribe the division of any bankrupt estate, provided there are just two creditors.

Here's an example: Suppose the estate consists of $125, and two creditors (call them Moe and Larry) claim $100 and $200. By the first principle, Larry's $200 claim is immediately reduced to $125. Now there is $100 in dispute and $25 undisputed. According to the contested garment principle, the disputed $100 is divided equally. Therefore Moe gets $50 and Larry gets the remaining $75.

Here's a second example: Suppose the estate consists of $125, and two creditors (call them Moe and Curly this time) claim $100 and $300. Curly's $300 claim is reduced to $125, and from there on, everything is just as in the first example. The bottom line: Moe gets $50 and Curly gets $75.

Here's a third example: Suppose the estate consists of $150, and two creditors (Larry and Curly) claim $200 and $300. Both claims are reduced to $150, leaving the entire $150 estate in dispute. So Larry and Curly get $75 each.

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But what should we do when there are three or more creditors—in other words, what should we do when Moe, Larry and Curly all make simultaneous claims against one estate? According to Professors Robert Aumann and Michael Maschler, of the Hebrew University in Jerusalem, we can solve this problem by introducing just one more principle, which they call *consistency*. According to the consistency principle, any pair of creditors must divide their collective share according to the principles we've already enunciated.

To see what consistency means in practice, think again about a $200 estate, against which Moe, Larry and Curly lay claims of $100, $200 and $300. The Talmud awards $50 to Moe and $75 to Larry, making their collective share $125. Their claims to that collective share are $100 and $200, so according to the first example above, the prescribed 50/75 division is exactly right.

At the same time, Moe and Curly lay claims of $100 and $300 to a collective share of $125, so according to the second example above, their prescribed 50/75 division is also exactly right. And Larry and Curly lay claims of $200 and $300 to a collective shareof $150, so according to the third example, their prescribed division—namely 75/75—is exactly right also.

That's what it means to say that the Talmudic prescription satisfies the consistency principle. But wait! All we've done is check that each pair of creditors divides its collective share appropriately; we haven't explained why (for example) Moe and Larry's collective share is $125 in the first place. Aumann and Maschler have an answer: Any division other than 50/75/75 would be inconsistent. (That is, with any other division, some pair of creditors would have its collective share divided incorrectly.)

In fact, they have proved more generally that every bankruptcy problem has *exactly one* consistent solution. Once you've found a consistent divison, you can be sure that no other is possible. So perhaps the Talmudists proceeded by trial and error, considering various divisions and rejecting each one as inconsistent until they hit upon one and only consistent division of 50/75/75. Or maybe they had a more systematic procedure.

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Here's a systematic procedure for generating the one and only consistent solution to any bankruptcy problem. Bear with me (or give up); I'm convinced there's no way to make this simpler.

Step I: If anybody claims more than the entire estate, reduce his claim to equal the entire estate.

Step II: Build (or at least draw) a "Talmudic jug" to represent the claims:
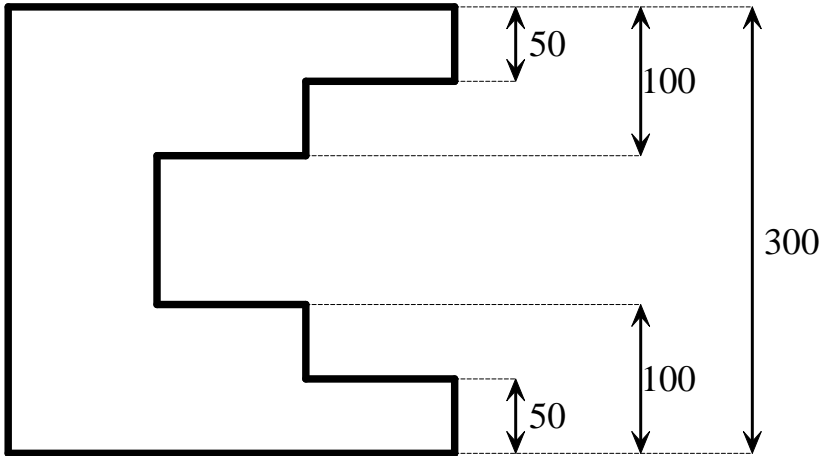
This particular jug represents Moe's, Larry's and Curly's claims of $100, $200 and $300, reading from right to left (in good Talmudic fashion). The combined height of the two rightmost compartments is 100 cubits, the combined height of the two middle compartments is 200 cubits, and the height of the leftmost compartment is 300 cubits. If your ruler doesn't show cubits, feel free to use inches or millimeters. It's important for all claims except the largest to be split into top and bottom compartments of equal size.

Here's a different jug, representing claims of $100, $300, $400 and $600:
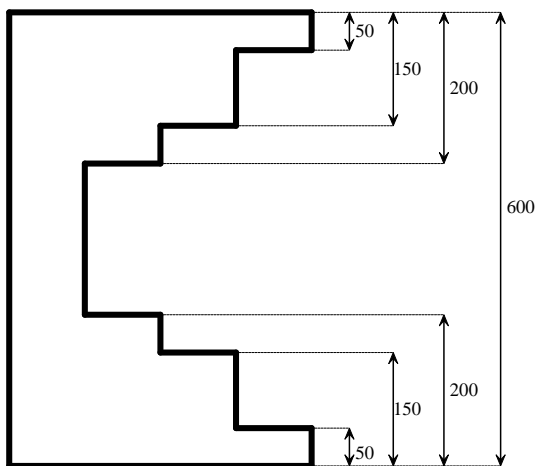
Step III: Pour in enough water to represent the estate. Here's the first jug with $200 worth of water (that is, enough to fill a container 200 cubits high):
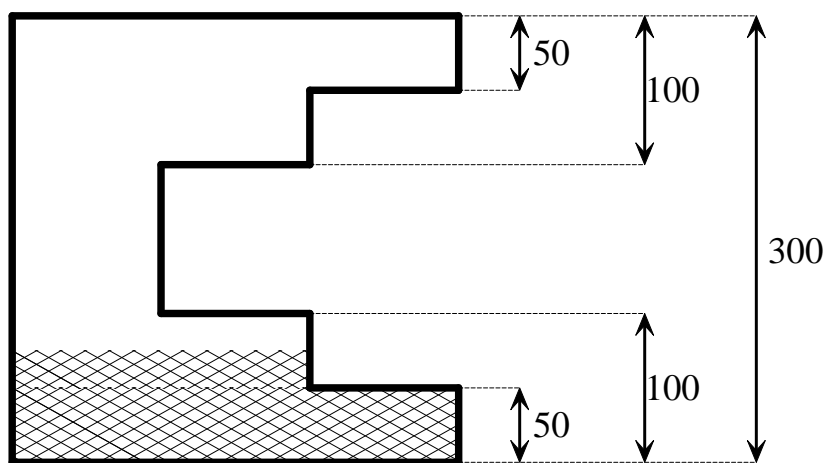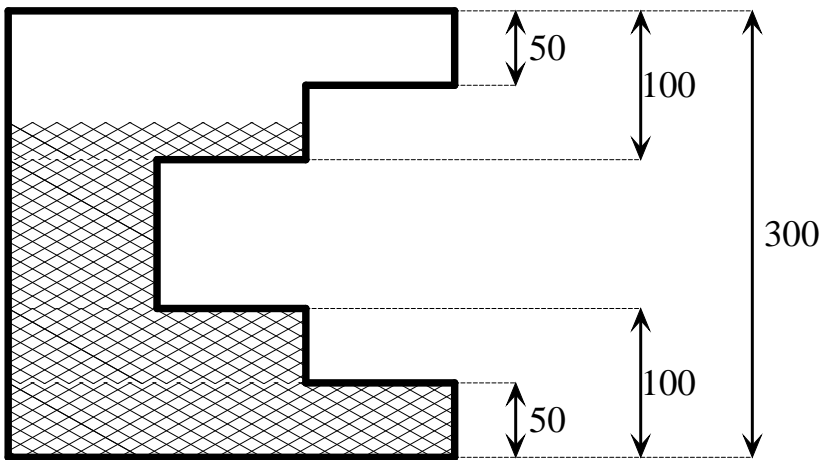
Step IV: Read off the payouts (again from right to left) according to the height of the water. In this case, Moe gets $50, because there are 50 cubits of water in the rightmost part of the jug. Larry gets $75, because there are 75 cubits in the middle part of the jug. Curly gets $75, because the water reaches a height of 75 cubits in the leftmost part of the jug.

So, according to the Talmudic jug, a split of 50/75/75 is the one and only consistent solution to this particular bankruptcy problem (that is, the problem of dividing a $200 estate among creditors with claims of $100, $200, and $300).

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What if Moe, Larry and Curly had laid their $100, $200 and $300 claims against an estate of $400? We could discover the consistent solution by filling the Talmudic jug with $400 worth of water:

Reading off the payouts from right to left, Moe gets $50, because there are 50 cubits of water in the rightmost part of the jug. Larry gets $125, because there are 125 cubits in the middle part of the jug (100 at Curly gets $225, because the water reaches a height of 225 cubits in the leftmost part of the jug.

Here's a quick check that the 50/125/225 division is really consistent (if you'd rather just take my word for it, feel free to skip this part!):

Moe and Larry assert claims of $100 and $200 to a collective share of $175. Larry's claim is reduced to $175. Now $100 is disputed and $75 is undisputed. According to the contested garment rule, the disputed $100 is divided equally. So Moe gets $50 and Larry gets $125, which agrees with the prescribed awards.

Moe and Curly assert claims of $100 and $300 to a collective share of $275. Curly's claim is reduced to $275. That leaves $100 in dispute and $175 undisputed. The disputed $100 is divided equally, so Moe gets $50 and Curly gets $225. Again, this agrees with the prescribed awards.

Larry and Curly assert claims of $200 and $300 to a collective share of $350. Thus Larry concedes $150 to Curly, Curly concedes $50 to Larry, and the remaining $150 is in dispute. According to the contested garment rule, the disputed $150 is divided equally ($75 to each), so Larry gets $50 + $75 = $125, and Curly gets $150 + $75 = $225. Again, this agrees with the prescribed awards.

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Although the ancient rabbis failed to consider this particular example (a $400 estate with claims of $100, $200 and $300), Aumann and Maschler express confidence that if they *had* considered it, they would have somehow discovered and endorsed this unique consistent solution.

Exactly *how* they'd have discovered it remains a mystery. The Talmudic jug is a thoroughly modern invention (in fact, it was invented by a *Slate* reader named Jayaraman Ramachandran to replace a less comprehensible presentation in one of my columns). It's very unlikely the rabbis could have known about it, so it's very unlikely they could have used it.

But whatever method the rabbis used, they appear to have used it (pardon the pun) consistently. It's not hard to check that every one of the Talmudic examples satisfies the consistency principle, which means that in every example they considered, the ancient Talmudists found the same answers as the modern jug-fillers. And the consistency principle gives a complete explanation for each example, in the sense that, in each case, only one consistent solution is possible, and we can imagine that the rabbis kept trying until they found it.

The consistency principle is both universally applicable (because a consistent solution can always be found) and universally unambiguous (because there is never more than one consistent solution).

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Why is the consistent solution the right solution? Aumann and Maschler argue that consistency appeals to our intrinsic sense of fairness. But, in the Talmudic tradition, if you don't like that argument, Aumann and Maschler have another.

Imagine that all the creditors are put in a room and told to agree among themselves on a division of the estate; if they can't agree, nobody gets anything. Suppose also that any creditor who is offered 100% of his claim (by a consensus of the others) is required

to accept it and leave the room. What would the bargaining process look like and what would the outcome be?

There is a branch of economics called "bargaining theory" that attempts to answer such questions; unfortunately the answers turn out to depend rather heavily on auxiliary assumptions. But Aumann and Maschler have proven that in the case of the bankruptcy negotiation, it follows from reasonable assumptions that the creditors would eventually agree to divide the estate in accordance with the consistency principle. Thus, according to Aumann and Maschler, all of the Talmudic prescriptions coincide with what the creditors themselves would have agreed to, given appropriate bargaining rules and sufficient time.