Generalized Class Field Theory

Kazuya Kato

Department of Mathematics, Faculty of Science
University of Tokyo, Hongo, Bunkyo-ku, Tokyo 113, Japan

This paper is a survey of the K-theoretic generalization of class field theory.

For a field K, let K^{ab} be a maximal abelian extension of K, that is, the union of all finite abelian extensions of K in a fixed algebraic closure of K. The classical local (resp. global) class field theory says that if K is a finite extension of the p-adic (resp. rational) number field $\mathbf{Q}_p$ (resp. $\mathbf{Q}$), the Galois group $\mathrm{Gal}(K^{\mathrm{ab}}/K)$ is approximated by the multiplicative group $K^\times$ (resp. the idele class group C_K), and via this approximation, we can obtain knowledge on abelian extensions of K.

In §1 (resp. §2), we give a K-theoretic generalization of the classical local (resp. global) class field theory. There finite extensions of $\mathbf{Q}_p$ (resp. $\mathbf{Q}$) are replaced by "higher dimensional local fields" (resp. finitely generated fields over prime fields), and the group $K^\times$ (resp. C_K) is replaced by Milnor's K-group $K_n^M(K)$ (resp. by the K_n^M-idele class group), where n is the "dimension" of K.

In §3, we discuss some other aspects of generalizations of local class field theory.

In §4, we discuss generalizations of the classical ramification theory to higher dimensional schemes.

1. Local Class Field Theory

An n-dimensional local field is defined inductively as follows. A 0-dimensional local field is a finite field. For $n \geq 1$, an n-dimensional local field is a complete discrete valuation field whose residue field is an $(n-1)$-dimensional local field.

For example, a finite extension of $\mathbf{Q}_p$ is a one dimensional local field.

For a field k, let $K_*^M(k)$ be Milnor's K-group of k defined by

$$K_q^M(k) = ((k^\times)^{\otimes q})/J$$

where J is the subgroup of the q-fold tensor product of $k^\times$ (as a $\mathbf{Z}$-module) generated by elements of the form $a_1 \otimes ... \otimes a_q$ such that $a_i + a_j = 1$ for some $i \neq j$. (Cf. Milnor [Mi].) The main result of the local class field theory of an n-dimensional local field is the following (Parsin [Pa$_2$], Kato [Ka$_1$] II).

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Theorem 1. *Let K be an n-dimensional local field. Then, there exists a canonical homomorphism*

$$\varrho : K_n^M(K) \longrightarrow \mathrm{Gal}(K^{\mathrm{ab}}/K)$$

which induces an isomorphism $K_n^M(K)/N_{L/K}K_n^M(L) \xrightarrow{\cong} \mathrm{Gal}(L/K)$ for each finite abelian extension L of K. Here $N_{L/K} : K_n^M(L) \to K_n^M(K)$ is the norm homomorphism ([Ka$_1$] II 1.7). *The correspondence $L \mapsto N_{L/K}K_n^M(L)$ is a bijection from the set of all finite abelian extensions of K in a fixed algebraic closure of K onto the set of all open subgroups of $K_n^M(K)$ of finite indices. (For the definition of the openness of a subgroup of $K_n^M(K)$, see* [Ka$_2$]*).*

2. Global Class Field Theory

Let X be a proper integral scheme over the ring of rational integers $\mathbf{Z}$ and let K be the function field of X. For simplicity we assume here $\mathrm{char}(K) = 0$ and that K has no ordered field structure.

For a non-zero coherent ideal I of $\mathcal{O}_X$ and for $q \geq 1$, define the sheaf of abelian groups $K_q^M(\mathcal{O}_X, I)$ on X_{zar} by

$$K_q^M(\mathcal{O}_X, I) = \mathrm{Ker}(K_q^M(\mathcal{O}_X) \to K_q^M(\mathcal{O}_X/I))$$

where

$$K_q^M(\mathcal{O}_X) = ((\mathcal{O}_X^\times)^{\otimes q})/J$$

with J the subgroup sheaf of the tensor product generated by local sections of the form $a_1 \otimes ... \otimes a_q$ such that $a_i + a_j = 1$ for some $i \neq j$, and $K_q^M(\mathcal{O}_X/I)$ is defined similarly. Define

$$C_I(X) = H^n(X_{\mathrm{zar}}, K_n^M(\mathcal{O}_X, I)), \quad \text{where} \quad n = \dim(X).$$

If I and I' are non-zero coherent ideals of $\mathcal{O}_X$, the inclusion $I \subset I'$ induces a surjection $C_I(X) \to C_{I'}(X)$. The main result of the class field theory of K is the following.

Theorem 2.1. *(1) $C_I(X)$ is a finite group for any I.*

(2) We have a canonical isomorphism of profinite abelian groups

$$\varprojlim_I C_I(X) \cong \mathrm{Gal}(K^{\mathrm{ab}}/K),$$

where I ranges over all non-zero coherent ideals of $\mathcal{O}_X$.

(3) For a non-empty regular open subscheme U of X, there exists a canonical isomorphism of profinite abelian groups

$$\varprojlim_I C_I(X) \cong \pi_1^{\mathrm{ab}}(U),$$

where I ranges over all non-zero coherent ideals of $\mathcal{O}_X$ such that $U \cap \mathrm{Spec}(\mathcal{O}_X/I) = \phi$, and $\pi_1^{\mathrm{ab}}(U)$ is the quotient of the algebraic fundamental group $\pi_1(U)$ of U modulo the closure of its commutator subgroup.

(4) If X is regular, then $C_{\mathcal{O}_X}(X)$ is isomorphic to the group $CH_0(X)$ of the classes of zero cycles on X modulo rational equivalence, and we have a canonical isomorphism of finite abelian groups

$$CH_0(X) \cong \pi_1^{\mathrm{ab}}(X).$$

An essential part of this theorem was proved by Bloch [Bl₁] in the case $\dim(X) = 2$. Cf. also [Pa₁] for the two dimensional case. The general case was proved in [KS₂] (see also S. Saito [SS₁]) by using the method in Bloch [Bl₁], except that we adopted in [KS₂] another definition of $C_I(X)$ which uses Nisnevich topology [Ni], a Grothendieck topology defined by Nisnevich [Ni] (called the henselian topology in [KS₂]), instead of Zariski topology. ($C_I(X) \overset{\mathrm{def}}{=} H^n(X_{\mathrm{Nis}}, K_n^M(\mathcal{O}_X, I))$ in [KS₂].). It was found later that Nisnevich topology and Zariski topology give the same $C_I(X)$ [KS₃].

The relation of the above theorem with the classical global class field theory is that if $X = \mathrm{Spec}(O_K)$ for a finite extension K of $\mathbf{Q}$, then $C_I(X)$ coincides with the ideal class group of conductor I of O_K.

The positive characteristic version of (4) of the above theorem is that if X is a proper smooth variety over a finite field k, $CH_0(X)^0 = \mathrm{Ker}(\deg : CH_0(X) \to \mathbf{Z})$ is finite and canonically isomorphic to $\mathrm{Ker}(\pi_1^{\mathrm{ab}}(X) \to \mathrm{Gal}(k^{\mathrm{ab}}/k))$. This fact was proved in [KS₁], and another proof was given in Colliot-Thélène, Sansuc and Soulé [CSS] and Gros [Gr]. That the former group surjects onto the latter was proved long ago by Lang whose paper [Lan] is the first work on the higher dimensional class field theory.

As an application of the generalized global class field theory, we have

Theorem 2.2. *For any scheme S of finite type over **Z**, the abelian group $CH_0(S)$ is of finite type.*

The case $\dim(S) = 2$ of this theorem was proved by Bloch [Bl₁], and the general case was proved in [KS₂].

Some finiteness theorems on $\pi_1^{\mathrm{ab}}(S)$ for arithmetic schemes S were proved in Katz-Lang [KL].

3. Some Aspects of the Generalized Local Class Field Theory

I give rough indications on some aspects of generalizations of local class field theory, which are not included in §1.

3.1 Explicit Reciprocity Law

The explicit reciprocity law for a finite extension of $\mathbf{Q}_p$ has been generalized to higher dimensional local fields of mixed characteristic by Vostokov-Kirrilov [VK] and Vostokov [V]. A generalization to complete discrete valuation fields of mixed characteristic $(0, p)$ with residue field F such that $[F : F^p] < \infty$ is given in [Ka4], as an application of the p-adic cohomology theory of Fontaine-Messing [FM].

Let K be "the most important two dimensional local field"

$$(\varprojlim_n (\mathbf{Z}/p^n\mathbf{Z})[[q]][q^{-1}]) \otimes_{\mathbf{Z}} \mathbf{Q}$$

where q is the q-invariant in the theory of moduli of elliptic curves. This field appears as a certain p-adic completion at infinity of a modular curve. As is shown in [Ka5], the explicit reciprocity laws of the two dimensional local fields $K(\zeta_{p^n})$, where ζ_{p^n} is a primitive p^n-th root of 1, are related to the special values of L-functions of elliptic modular forms and to an Iwasawa theory of elliptic modular forms, just as the explicit reciprocity laws of $\mathbf{Q}_p(\zeta_{p^n})$ are related to the special values of Riemann zeta function and to the classical Iwasawa theory.

3.2 Semi-Global Theories

The class field theory of curves over (usual) local fields was studied by Coombes [Co] and S. Saito [SS2], and that of surfaces over local fields was studied recently by S. Saito and Salberger.

The class field theory of complete discrete valuation fields whose residue fields are (usual) global fields of positive characteristic was sought for first by Ihara [Ih], and studied by [Ka1] III.

The class field theory of two dimensional complete noetherian local rings with finite residue fields (for example $\mathbf{Z}_p[[T]]$) was studied by S. Saito [SS3].

3.3 Serre's Local Class Field Theory

Serre's local class field theory of a complete discrete valuation field with algebraically closed residue field [Se2], and its generalization by Hazewinkel to the perfect residue field case [Ha], are generalized to the imperfect residue field case as a duality theorem of the following form: If K is a complete discrete valuation field with residue field F such that $\mathrm{char}(F) = p > 0$ and $[F : F^p] = p^r < \infty$, then the Galois cohomologies

$$(*) \qquad R\Gamma(K, \mathbf{Z}/p^n\mathbf{Z}(s)) \quad \text{and} \quad R\Gamma(K, \mathbf{Z}/p^n\mathbf{Z}(t)), \qquad s + t = r + 1$$

are in perfect duality via the dualizing functor $R\,\mathrm{Hom}_{\mathbf{Z}/p^n\mathbf{Z}}(\ , W_n\Omega^r_{F,\log})[r+1]$. Here the objects $(*)$ and the logarithmic part of the de Rham-Witt sheaf $W_n\Omega^r_{F,\log}$ ([Il]) are not regarded just as a complex of abelian groups or as objects on the small etale site on $\mathrm{Spec}(F)_{\mathrm{et}}$, but regarded as objects on a much bigger site. (Precisely speaking, they are regarded as objects on the site of schemes S over

F which are locally isomorphic to "relative perfections" ([Ka3] I, II) of smooth schemes over F, endowed with the etale topology.) The details of this duality theorem are given in [Ka3] III.

For example, if F is an algebraically closed field, this duality and the inclusion

$$U_K/(U_K)^{p^n} \subset K^\times/(K^\times)^{p^n} = H^1(K, \mathbf{Z}/p^n\mathbf{Z}(1))$$

induces $H^1(K, \mathbf{Z}/p^n\mathbf{Z}) \cong \mathrm{Ext}^1(U_K, \mathbf{Z}/p^n\mathbf{Z})$, where U_K is the unit group of K which is not regarded just as a group, but regarded as a pro-algebraic group over F. This reproduces the p-primary part (the essential part) of Serre's local class field theory

$$\pi_1(U_K) \cong \mathrm{Gal}(K^{\mathrm{ab}}/K).$$

4. Ramification Theory

In the classical ramification theory, we mainly consider a finite extension B/A of discrete valuation rings with perfect residue fields. We have the following three kinds of important invariants of ramification: The different $\delta(B/A) \in \mathbf{Z}$; in the case of Galois extension with Galois group G, the Lefschetz numbers

$$i(\sigma) = \mathrm{length}(B/I_\sigma) \in \mathbf{Z} \quad \text{for} \quad \sigma \in G - \{1\},$$

with I_σ the ideal of B generated by $\{\sigma(b) - b\,;\, b \in B\}$; also in the Galois case, the Artin (or Swan) conductors of representations of G. Is it possible to generalize these invariants and relations between them such as

$$(*) \qquad \delta(L/K) = \sum_{\sigma \in G - \{1\}} i(\sigma)$$

to higher dimensional schemes?

Concerning $\delta(B/A)$ and $i(\sigma)$, Bloch obtained in [Bl2] a nice ramification theory in higher dimensions, generalizing $\delta(B/A)$ and $i(\sigma)$ to zero cycle classes on the ramification locus (i.e. to elements of $CH_0(E)$ where E is the ramification locus). His projection formula [Bl2] (7.1) is an extension of (*) to two dimensional schemes.

Concerning conductors, there are many different attempts of generalizations (Deligne [De], Laumon [La], S. Saito [SS4], Berthelot [Be1, Be2]). In the following, I only introduce my method on the conductors of one dimensional Galois representations [Ka7] which is closely related to the generalized local class field theory. I generalize, under the influence of Bloch's theory [Bl2], the Swan conductors to zero cycle classes on the wild ramification locus in the two dimensional case (the plan exists even for dimension ≥ 3), and give applications Theorems (4.1)–(4.3).

Let X be an excellent connected normal scheme, U a regular dense open subscheme of X, and let $\chi : \pi_1^{\mathrm{ab}}(U) \to \Lambda^\times$ be a continuous homomorphism where Λ is a discrete (commutative) field. We are interested in the wild ramification of χ

on X. We assume that the wild ramification locus E of χ on X (with the reduced scheme structure) is a disjoint union of schemes of finite type over perfect fields. This condition is satisfied for example if X is of finite type over $\mathbf{Z}$ or over a perfect field.

First, we have a divisor called "Swan conductor divisor" ([Ka$_6$]) $\mathrm{sw}(\chi) = \sum_{\mathfrak{p}} \mathrm{sw}_{\mathfrak{p}}(\chi)\,\overline{\{\mathfrak{p}\}}$ on X ($\overline{\{\mathfrak{p}\}}$ denotes the closure of $\{\mathfrak{p}\}$), where $\mathfrak{p}$ ranges over all points of codimension one. The integer $\mathrm{sw}_{\mathfrak{p}}(\chi)$ has the following properties: (i) If $\dim(X) = 1$, it coincides with the classical Swan conductor of χ at $\mathfrak{p}$. (ii) $\mathrm{sw}_{\mathfrak{p}}(\chi) > 0$ if and only if χ is wildly ramified at $\mathfrak{p}$. Furthermore, if the following condition

(C) X is regular and $D = (X - U)_{\mathrm{red}}$ is a divisor with normal crossings on X,

we have a canonical global section

$$\mathrm{rsw}(\chi) \in \Gamma(E,\ \mathcal{O}_E \otimes_{\mathcal{O}_X} \Omega^1_X(\log(D)) \otimes_{\mathcal{O}_X} \mathcal{O}_X(\mathrm{sw}(\chi)))$$

called the refined Swan conductor ([Ka$_7$] I). Here $\Omega^1_X(\log(D))$ is the $\mathcal{O}_X$-module on X_{et} defined by generators $\mathrm{dlog}(a)$ $(a \in j_*(\mathcal{O}_U^\times)$ with j the inclusion map $U \to X)$ subject to natural relations such as "$a\mathrm{dlog}(a)$ is additive in a", and $\mathcal{O}_X(\mathrm{sw}(\chi))$ denotes the invertible $\mathcal{O}_X$-module corresponding to the divisor $\mathrm{sw}(\chi)$. In this case, E coincides with the support of the divisor $\mathrm{sw}(\chi)$. It can be shown that the $\mathcal{O}_E$-module $\mathcal{O}_E \otimes_{\mathcal{O}_X} \Omega^1_X(\log(D))$ is locally free of rank (locally) $\dim(E)+1$. If X is of finite type over $\mathbf{Z}$, these $\mathrm{sw}(\chi)$ and $\mathrm{rsw}(\chi)$ have explicit descriptions in terms of K-theoretic class field theory ([Ka$_7$] I). We say (X, U, χ) is clean if at any point x of E, the stalk of $\mathrm{rsw}(\chi)$ at x (which is always non-zero) is a part of a basis of $\mathcal{O}_E \otimes_{\mathcal{O}_X} \Omega^1_X(\log(D)) \otimes_{\mathcal{O}_X} \mathcal{O}_X(\mathrm{sw}(\chi))$ at x. If (X, U, χ) is clean, we define a cycle $\mathrm{Char}(X, U, \chi)$ on the vector bundle $\mathcal{O}_E \otimes_{\mathcal{O}_X} \Omega^1_X(\log(D))$ over E by

$$\mathrm{Char}(X, U, \chi) = \sum_{\mathfrak{p}} \mathrm{sw}_{\mathfrak{p}}(\chi)\,\mathrm{Image}(\varphi_{\mathfrak{p}}),$$

where $\varphi_{\mathfrak{p}}$ is the map $\mathcal{O}_E \otimes_{\mathcal{O}_X} \mathcal{O}_X(-\mathrm{sw}(\chi)) \to \mathcal{O}_E \otimes_{\mathcal{O}_X} \Omega^1_X(\log(D))$ induced by $\mathrm{rsw}(\chi)$, and $\mathrm{Image}(\varphi_{\mathfrak{p}})$ is regarded as a subbundle of $\mathcal{O}_E \otimes_{\mathcal{O}_X} \Omega^1_X(\log(D))$. In the clean case, we define our generalization of the Swan conductor

$$c(X, U, \chi) \in CH_0(E)$$

to be $(-1)^{\dim(E)}$ times the intersection class of the two cycles $\mathrm{Char}(X, U, \chi)$ and the zero section of the vector bundle $\mathcal{O}_E \otimes_{\mathcal{O}_X} \Omega^1_X(\log(D))$. This $\mathrm{Char}(X, U, \chi)$ is an analogue of the charateristic cycle in the theory of $\mathcal{D}$-modules.

Now without the condition (C), I conjecture that there exists a proper birational morphism $f : X' \to X$ such that $f^{-1}(U) \xrightarrow{\cong} U$ and $(X', f^{-1}(U), \chi)$ is clean. It can be proved ([Ka$_7$] I) that if $\dim(X) \le 2$, this conjecture is true and

$$c(X, U, \chi) \stackrel{\mathrm{def}}{=} f_* c(X', f^{-1}(U), \chi) \in CH_0(E)$$

is independent of the choice of such X'.

Theorem 4.1. *Let X be a proper normal surface over an algebraically closed field k, U a regular dense open subscheme of X, ℓ a prime number different from* $\mathrm{char}(k)$, *$\chi : \pi_1^{\mathrm{ab}}(U) \to \overline{\mathbf{F}}_\ell^\times$ a continuous homomorphism, and $\mathscr{F}$ the $\overline{\mathbf{F}}_\ell$-sheaf on U_{et} of rank one corresponding to χ. Then,*

$$\chi(U, \mathscr{F}) = \chi(U, \overline{\mathbf{F}}_\ell) - \deg(c(X, U, \chi)).$$

Here $\chi(\ )$ denotes the Euler-Poincaré characteristic $\sum_i (-1)^i \dim_{\overline{\mathbf{F}}_l} H^i_{\mathrm{et}}(\ )$.

This theorem was generalized by T. Saito [ST$_2$] to the case $\dim(X)$ is arbitrary under a certain assumption on χ .

The Riemann-Roch formula for the Euler-Poincaré characteristics of ℓ-adic sheaves ($\ell \neq$ characteristic), which should generalize the Grothendieck-Ogg-Shafarevich formula on ℓ-adic sheaves on curves to higher dimensional varieties, was sought for first by Grothendieck. A formula in the characteristic zero case was obtained by MacPherson [Ma]. In the positive characteristic case, results for surfaces, of types different from (4.1), have been obtained by Deligne [De], Laumon [Lau] (using ideas of Deligne), S. Saito [SS$_4$]. Deligne is the first person who had the idea of the characteristic cycle of an ℓ-adic sheaf. Recently, Berthelot obtained a Riemann-Roch formula for $\mathscr{D}$-modules with Frobenius in characteristic p by defining the characteristic cycle of a $\mathscr{D}$-module with Frobenius ([Be$_2$] basing on his theory [Be$_1$]). There should be close relations between his characteristic cycle, the characteristic cycle of Deligne, and the characteristic cycle $\mathrm{Char}(X, U, \chi)$ discussed above.

Theorem 4.2. *Let A be a complete discrete valuation ring with field of fractions k and with perfect residue field F. Let X be a regular connected A-scheme which is proper flat of relative dimension one over A, U a dense open subscheme of $X_k = X \otimes_A k$, let ℓ be a prime different from* $\mathrm{char}(F)$, *let $\chi : \pi_1^{\mathrm{ab}}(U) \to \overline{\mathbf{F}}_\ell^\times$ be a continuous homomorphism, and let $\mathscr{F}$ be the $\overline{\mathbf{F}}_\ell$-sheaf of rank one on U corresponding to χ . Assume χ is at worst tamely ramified at points in $X_k - U$. (This last condition is satisfied automatically if* $\mathrm{char}(k) = 0$*). Then*

$$\mathrm{sw}(R\Gamma((U_{\bar{k}})_{\mathrm{et}}, \mathscr{F})) = \mathrm{sw}(R\Gamma((U_{\bar{k}})_{\mathrm{et}}, \overline{\mathbf{F}}_\ell)) + \deg(c(X, U, \chi)).$$

Here $\mathrm{sw}(R\Gamma(\ )) = \sum_i (-1)^i \mathrm{sw}(H^i(\ ))$, with sw the Swan conductor of a representation of $\mathrm{Gal}(k^{\mathrm{sep}}/k)$.

In [Bl$_2$], Bloch obtained a formula which expresses $\mathrm{sw}(R\Gamma((U_{\bar{k}})_{\mathrm{et}}, \overline{\mathbf{F}}_\ell))$ in terms of the differential module $\Omega^1_{X/A}$.

Finally we discuss a conjecture of Serre. Let B be a regular local ring and let G be a finite subgroup of $\mathrm{Aut}(B)$. Assume that the following conditions (i)(ii) are satisfied.

(i) For any $\sigma \in G - \{1\}$, B/I_σ is of finite length where I_σ denotes the ideal of B generated by $\{\sigma(b) - b\ ;\ b \in B\}$.

(ii) Let $A = \{b \in B\ ;\ \sigma(b) = b \quad \text{for any} \quad \sigma \in G\}$. Then, A is noetherian and $A/m_A \to B/m_B$ is an isomorphism (m_* denote the maximal ideals).

Define the function $a_G : G \to \mathbf{Z}$ by

$$a_G(\sigma) = -\operatorname{length}(B/I_\sigma) \quad \text{for} \quad \sigma \in G - \{1\},$$

$$a_G(1) = -\sum_{\sigma \in G - \{1\}} a_G(\sigma).$$

If $\dim(B) = 1$, a_G is a character of a representation of G called Artin representation. Serre conjectures that a_G is a character of a representation of G even when $\dim(B) > 1$ ([Se_1]).

Theorem 4.3. *The conjecture is true if* $\dim(B) = 2$.

This theorem was proved in Kato, S. Saito, T. Saito [KSS] in the equal characteristic case, and in T. Saito [ST_1] in the mixed characteristic case under a certain assumption.

The outline of the proof of (4.3) given in [Ka_7] II is very similar to the proof for the one dimensional case. It is sufficient to prove that if χ is the character of a representation of G, then

$$\operatorname{Card}(G)^{-1} \sum_{\sigma \in G} a_G(\sigma)\chi(\sigma) \in \mathbf{Z}.$$

By using the two dimensional version in Bloch [Bl_2] of the formula (*) at the bigining of §4 and by the theory of Brauer, just as in the one dimensional case, we are reduced to the case χ is of degree one. We may assume that A is complete, A/m_A is perfect, and χ is wildly ramified. Let $X = \operatorname{Spec}(A)$, $E = \operatorname{Spec}(A/m_A) \subset X$ and $U = X - E$. Then, we can prove

$$\operatorname{Card}(G)^{-1} \sum_{\sigma \in G} a_G(\sigma)\chi(\sigma) = 1 + \deg(c(X, U, \chi)) \in \mathbf{Z}$$

where deg is the canonical isomorphism $CH_0(E) \xrightarrow{\cong} \mathbf{Z}$.

References

[Be_1] Berthelot, P.: Cohomologie rigid et théorie des $\mathscr{D}$-modules. Preprint

[Be_2] Berthelot, P.: In preparation

[Bl_1] Bloch, S.: Algebraic K-theory and class field theory for arithmetic surfaces. Ann. Math. **114** (1981) 229–266

[Bl_2] Bloch, S.: Cycles on arithmetic schemes and Euler characteristics of curves. Proc. Symp. Pure Math. AMS **46** Part 2 (1987) 421–450

[CSS] Colliot-Thélène, J.-L., Sansuc, J.-J., Soulé, C.: Torsion dans le groupe de Chow de codimension deux. Duke Math. J. **50** (1983) 763–801

[Co] Coombes, K. R.: Local class field theory for curves. Contemp. Math. **55** I (1983) 117–134

[De] Deligne, P.: Letter to Illusie, Nov. 4, 1976

[FM] Fontaine, J. M., Messing, W.: p-adic periods and p-adic etale cohomology. Contemp. Math. **67** (1987) 179–207

[Fu] Fulton, W.: Intersection theory. Springer, Berlin Heidelberg New York 1984

[Gr] Gros, M.: Sur la partie p-primaire du groupe de Chow de codimension deux. Comm. Alg. **13** (1985) 2407–2420

[Ha] Hazewinkel, M.: Corps de classes local, Appendix to Demazure, M., Gabriel, P.: Groupes algébriques, North-Holland Publ. 1970, pp. 648-674

[Ih] Ihara, Y.: On a problem on some complete p-adic function fields (in Japanese). Kokuroku 41, RIMS, Kyoto Univ. (1968) 7–17

[Il] Illusie, L.: Complexe de de Rham-Witt et cohomologie cristalline. Ann. Sci. Ec. Norm. Sup. **12** (1976) 501–661

[Ka_1] Kato, K.: A generalization of local class field theory by using K-groups, I., J. Fac. Sci. Univ. of Tokyo, Sec. IA, **26** (1979) 303–376; II, ibid. **27** (1980) 603–683; III, ibid. **29** (1982) 31–43

[Ka_2] Kato, K.: Class field theory and algebraic K-theory. (Lecture Notes in Mathematics, vol. 1016.) Springer, Berlin Heidelberg New York, pp. 109–127

[Ka_3] Kato, K.: Duality theories for p-primary etale cohomology. Algebraic and topological theories. I, Kinonuniya (1985) 127–148; II, Comp. Math. **63** (1987) 259–270; III. In preparation

[Ka_4] Kato, K.: The explicit reciprocity law and the cohomology of Fontaine-Messing. Preprint

[Ka_5] Kato, K.: p-adic Hodge theory and values of zeta functions of elliptic cusp forms. In preparation

[Ka_6] Kato, K.: Swan conductors for characters of degree one in the imperfect residue field case. Contemp. Math. **83** (1989) 101–132

[Ka_7] Kato, K.: Class field theory, $\mathscr{D}$-modules, and ramification on higher dimensional schemes. I, to appear in Amer. J. Math.; II. In preparation

[KL] Katz, N., Lang, S.: Finiteness theorems in geometric class field theory. l'Enseignement Mathématique **27** (1981) 285–319

[KS_1] Kato, K., Saito, S.: Unramified class field theory of arithmetical surfaces. Ann. Math. **118** (1983) 241–275

[KS_2] Kato, K., Saito, S.: Global class field theory for arithmetic schemes. Contemp. Math. **5** Part I (1986) 255–330

[KS_3] Kato, K., Saito, S.: In preparation

[KSS] Kato, K., Saito, S., Saito, T.: Artin characters for algebraic surfaces. Amer. J. Math. **110** (1988) 49–76

[Lan] Lang, S.: Unramified class field theory over function fields in several variables. Ann. Math. **64** (1956) 286–325

[Lau] Laumon, G.: Caractéristique d'Euler Poincaré des faisceaux constructibles sur une surface. Asterisque **101–102** (1983) 193–207

[Ma] MacPherson, R.: Chern classes of singular varieties. Ann. Math **100** (1974) 423–432

[Mi] Milnor, J.: Algebraic K-theory and quadratic forms. Inv. math. **9** (1970) 318–344

[Ni] Nisnevich, A.: Arithmetic and cohomology invariants of semi-simple group schemes and compactifications of locally symmetric spaces. Funct. Anal. Appl. **14**, no. 1 (1980) 61–62

[Pa_1] Parsin, A. N.: Abelian coverings of arithmetic schemes. Sov. Math. Dokl. **19**, no. 6 (1978) 1438–1442

[Pa_2] Parsin, A. N.: Local class field theory. Proc. Steklov Inst. Math. **165** (1985) 157–185

[Se_1] Serre, J.-P.: Sur la rationalité des représentations d'Artin. Ann. Math. **72** (1960) 406–420

[Se_2] Serre, J.-P.: Sur les corps locaux à corps résiduel algébriquement clos. Bull. Soc. Math. France **89** (1961) 105–154

[SS_1] Saito, S.: Unramified class field theory for arithmetic schemes. Ann. Math. **121** (1985) 251–281

[SS_2] Saito, S.: Class field theory for curves over local fields. J. Number Theory **21** (1985) 44–80
[SS_3] Saito, S.: Class field theory for two dimensional local rings. Adv. Stud. Pure Math. **12** (1987) 343–373
[SS_4] Saito, S.: General fixed point formula for an algebraic surfaces and the theory of Swan represenations for two dimensional local rings. Amer. J. Math. **109** (1987) 1009–1042
[ST_1] Saito, T.: Bloch's 0-cycles and Artin characters of arithmetic surfaces. Preprint (1986)
[ST_2] Saito, T.: The Euler numbers of ℓ-adic sheaves of rank 1 in positive characteristic. Preprint
[Vo] Vostokov, S. V.: Explicit construction of class field theory for a multidimensional local field. Math. USSR Izv. **26** (1986) 263–287
[VK] Vostokov, S. V., Kirillov A. N.: Normed pairing in a two dimensional local field. J. Sov. Math. **30**, no. 1 (1985) 1847–1853